A Formal Perspective on IEC 61499 Execution Control Chart Semantics

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Background

Well-Formed Execution Control Charts Coq Formalization Conclusion / Future Work IEC 61499 Formal methods

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IEC 61499 Formal methods



IEC 61499 Models

What is IEC 61499?

- A model for loosely coupled distributed systems.
- Component Based (Function Blocks)
- Asynchronous Events with Event/Data association.
- Function Block *networks* mapped to *resources*.
- Resources mapped to devices.

International Standard IEC 61499: Function Blocks - Part 1, Architecture, Geneva, Switzerland: Int. Electrotech. Commission, 2012.

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IEC 61499 Formal methods



Does your model implement the intended behavior?

Two sides of the problem:

- **Model-level** verification
 - Well-formedness (soundness)
 - Intended behavior
- 2 Tool-chain verification
 - Analysis, e.g. well-formedness check
 - Compilation & Deployment
 - Run-time systems & Networking

Verification needs a formal underpinning!

IEC 61499



Contributions

Our contributions in short:

- Semantics of IEC 61499 (sub-set) formalized in Coq
- Well-formedness criterion for scheduling progression
- Graph-based methods for static (compile-time) analysis
- Methods implemented in Coq (not yet proven)
- A prototype implementation based on extracted code

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Does your model implement the intended behavior?

Related to the original problem:

- **Model-level** verification
 - Well-formedness (w.r.t scheduling progression)
 - Intended behavior
- 2 Tool-chain verification
 - Analysis (w.r.t scheduling progression)
 - Compilation & Deployment
 - Run-time systems & Networking

We provide a formal underpinning for verification

IEC 61499 Formal methods



Design Elements, Function Block Interface

Function Block Interface:

- Events Input and output events,
- Variables Input, output, and local variables, and
 - With Association between events and data.



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Design Elements, Function Block Type

Function Block Types:

- BFB Basic Function Blocks used to specify general behavior,
- SIFB Service Interface Function Blocks used to interface the environment of a FB network,
- CFB Composite Function Blocks composition of BFBs/SIFBs and (inner) CFBs mapped as a single element for deployment, and
- SUB Sub-application
 - composition of BFBs/SIFBs/CFBs and (inner) SUBs each inner element mapped separately.

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Design Elements, Basic Function Block

- Used to specify stateful behaviour,
- Each *state* may be associated to a sequence of *actions*. An *action* is defined by:
 - An (optional) algorithm
 - An (optional) output event



IEC 61499 Formal methods



Formalization

The standard gives an *informal* specification of the IEC 61499 semantics. In literature we find numerous approaches to formalization, including:

- V. Vyatkin, Execution Semantic of Function Blocks based on the Model of Net Condition/Event System, in Industrial Informatics, 2006 IEEE International Conference on, Aug 2006)
- V. Dubinin and V. Vyatkin, *On Definition of a Formal Model* for *IEC 61499 Function Blocks*, EURASIP J. Embedded Syst., vol. Apr. 2008.
- G. Cengic and K. Akesson, *On Formal Analysis of IEC 61499 Applications, Part A: Modeling*, IEEE Transactions on Industrial Informatics, vol. 6, no. 2, 2010.

IEC 61499 Formal methods



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Formal methods

Different methods to verification:

- Model checking
 - Define (some) property of the model
 - (+) Automatic checking
 - (-) May lead to state explosion
 - (-) May need to re-check whole model, even on subtle change
- Deductive reasoning
 - Define (some) property of the model & prove obligation(s)/goal(s)
 - (-) Manual or Semi-automatic
 - (+) Once proven, holds forever!
 - (+) Re-use of lemmas
 - (+) Tools may allow for extraction of *certified* code

IEC 61499 Formal methods



Tools for Deductive reasoning

- **Coq** (INRIA) is a theorem-based proof assistant:
 - $\bullet\,$ Definitions are given in a typed $\lambda\text{-calculus that features:}\,$
 - polymorphism,
 - dependent types and
 - very expressive (co-)inductive types
 - Proofs are done *semi-automatic* (through applying tactics)
 - Proofs are automatically checked
- why3 (INRIA) is an extension to Hoare logic:
 - derives proof obligations from pre- and post-conditions
 - interfaces to (1st order logic) *automatic* provers, e.g. Alt-Ergo, CVC3/CVC4, Spass, Z3, etc.
 - can also export definitions and goals to Coq (in case automatic methods does not succeed)

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Notation ECC Execution Semantics ECC liveness conditions

Formalized

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Execution Control Chart

The ECC specification is defined as a graph:

 $ECC \triangleq \langle Q, T \rangle,$

where Q is a finite set of ECC states $q \in Q$, and T is the finite set of arcs or transitions $t \in T$. A transition $t \in T$

A transition $t \in T$ is defined as the triple

 $t = \langle q_s, c, q_d \rangle,$

where q_s and q_d , the source/destination state, and c a Boolean guard condition encoded via the functional signature,

$$c: e_i \times D_i \times D_o \times D_l \rightarrow Bool,$$

where $e_i \in E_i$

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Notation ECC Execution Semantics ECC liveness conditions



Execution Control Chart

BFB-states $s \in S$ are quadruples of the form $\langle d_i, d_o, d_l, q \rangle$. The initial state in more detail,

$$S^0 \triangleq \langle d_i^0, d_o^0, d_l^0, q^0 \rangle,$$

where $d_i^0 \in D_i$, $d_o^0 \in D_o$, and $d_l^0 \in D_l$ are input, output, and local data variables, respectively, and q^0 defines the initial *ECC* state

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ECC Execution Semantics

The standard defines the "ECC operation state machine":

s0 Idle (initial) state,

- s1 evaluate transisitons,
- s2 execute actions,
- t1 on event sample data,
- t3 on guard expression *true* cross transition,
- t4 on all actions executed, and
- t2 on all guard expressions false



Figure: *ECC_{ex}* state machine behavior

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ECC Execution Semantics

ECC Execution Semantics

Quoting the standard:

- 1 ... the resource shall ensure that no more than one input event occurs at any given instant in time ...:
- 2 ... Algorithm execution in a basic function block shall consist of the execution of a finite sequence of operations ...;
- In the state s1 was entered via t1, only transition conditions associated with the current input event, or transition conditions with no event associations, shall be evaluated. If state s1 was entered via t4, only transition conditions with no event associations shall be evaluated ・ロト ・ 一下・ ・ 日 ・ ・ 日 ・



Figure: ECC_{ex} state machine behavior

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ECC liveness conditions

Liveness is a common property to all well-formed models, and specifies that at some point *progression* is ensured

In our case we define liveness by scheduling progression

We discriminate between:

- well-formed models that ensures scheduling progression
- *ill-formed* models that do not ensure scheduling progression

Key observation:

Only on transition t1 (from s0) new events are received

Notation ECC Execution Semantics ECC liveness conditions



How to ensure progression?

 ECC_{ex} must (eventually) reach state s0 to accept a new event:

- On ECC_{ex} invocation $s0 \xrightarrow{t1} s1$ is taken.
- The transition conditions s1 of ECC state q_n lead either to:
 - transition $s1 \stackrel{t2}{\rightarrow} s0$ and consequent *liveness*, or
 - transition $s1 \stackrel{t3}{\rightarrow} s2$ and action execution
 - statement 2 (finite sequence of operations), ensures termination of s2, thus: checking that s1 ^{t2}/_→ s0 is eventually taken is a sufficient and necessary liveness criterion, seen as a function:

$$\forall q_n, e, ECC_{ex}(ECC, q_n, e) \stackrel{\star}{\rightarrow} s0,$$

where ECC is the ECC graph, q_n any state and e any event

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Necessary and Sufficient Liveness Condition

Theorem (Necessary and Sufficient Liveness Condition)

If each edge in the ECC is crossed a bound number of times, then s1 $\stackrel{t2}{\rightarrow}$ s0 will eventually be taken.

Ensuring this is en general hard! It involves proving termination condition t^2 under arbitrary algorithms (and their side effects to local variables d_l and output variables d_o)

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Sufficient Liveness Condition

Theorem (Sufficient Liveness Condition)

If each edge in the ECC is crossed at most one time, then $s1 \xrightarrow{t2} s0$ will eventually be taken.

Limits expressivity, (we do not allow arbitrary loops in the ECC) However:

The IEC 61499 standard stipulates, statement 3:

...) If state s1 was entered via t1, only transition conditions associated with the current input event, or transition conditions with no event associations, shall be evaluated. If state s1 was entered via t4, only transition conditions with no event associations shall be evaluated (...).

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Sufficient Liveness Condition

We can now formulate a sufficient (safe) condition:

- Let $ev(t) : T \rightarrow Bool$ be a mapping from a transition t to true if the corresponding guard condition from the respective ECC holds an event dependency
- Let the function *SCC*(*ECC*) result in the set of strongly connected components (sub-graphs) of the *ECC* The following generalization is possible:

$$\forall scc \in SCC(ECC), \exists t \in scc, ev(t) = true,$$

i.e., each cyclic path must have at least one edge for which the guard involves an event (i.e., ev(t) holds)

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Example: Well-formed ECC (1/2)

Well-formed ECC (ECC_{wf}): Green arrow indicate a transition t, where ev(t) = true.



Figure: *ECC_{wf}*.

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Example: Well-formed ECC (2/2)

Well-formed ECC (ECC_{wf}) :

Green arrow indicate a transition t, where ev(t) = true.



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Example: Ill-formed ECC

Ill-formed ECC (*ECC_{ill}*):

Red cycles indicate an ill-formed transition chain.



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ECC scheduling progression, alternative formulation

- From graph theory, is known that for any directed graph, the set of *maximal* SCC can be derived in linear time.
- A maximal SCC may have inner SCCs, thus we need to enumerate and check $v_i \stackrel{\star}{\rightarrow} v_j$ and $v_j \stackrel{\star}{\rightarrow} v_i$, $(v_i, v_j \in scc)$.
- However (related) the enumeration of *minimal SCCs*, is known to be NP complete.
- We can turn the problem into a pre-processing alternate by applying ev(t) to the ECC prior to deriving the corresponding *SCCs*. Let us define, as follows:

$$\textit{ECC}^{\textit{pre}} = \textit{ECC} \setminus \{t \in \textit{ECC} \mid \textit{ev}(t) = \textit{true}\}$$

Well-formedness can now be formulated as the following set emptyness check:

$$SCC(ECC^{pre}) = \{\emptyset\}$$

ECC liveness conditions



Example: Pre-processing of well-formed ECC

The example $SCC(ECC_{wf}^{pre}) = \{\emptyset\}$, i.e., ECC^{pre} has no strongly connected components (cycles).



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Example: Pre-processing of ill-formed ECC

The example $SCC(ECC_{ill}^{pre}) \neq \{\emptyset\}$, i.e., ECC^{ill} has a strongly connected component (cycle).



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Coq Definitions Extraction Integration



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Coq Definitions Extraction Integration



Coq Formalization



- Computational definitions can be *proven* and extracted to *certified* functional code
- Realistic sized programs: CompCert C
- However it is not easy (CompCert C > 10 years)
- Our work, just a proof of concept

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Coq Definitions Extraction Integration



Coq Definitions : guard

The Basic Function Block (BFB) notations can be captured by record types and plain definitions in Coq.

As an example, the definition of the transition guard expression.

```
1 Definition nodeId_t := nat.
2 Definition eventId_t := nat.
3 
4 Record guard_t := mkGuard {
5 onEvent : option eventId_t;
6 onExp : bool
7 }.
```

Listing 1: Coq definitions (excerpt).

This is a simplification, considering boolean guard expression: onExp : d_i \rightarrow d_l \rightarrow d_o \rightarrow bool

Coq Definitions Extraction Integration



Coq Definitions : clear

The *computational* evaluation function clear takes an event eid and a guard expression guard and evaluates to (true false).

```
Definition guard_target_t := prod guard_t nodeId_t.
 1
 2
    Definition edge t
                                := prod nodeId t guard target t.
 3
    Definition node t
                                := list action t.
    Definition nodes t
                               := list (prod nodeId_t node_t).
 4
 5
    Definition edges t
                               := list edge_t.
6
 7
    (* Checks if guard expression is true *)
 8
    Definition clear (eid:eventId t) (guard:guard t) :=
9
      let cEvent :=
10
      match onEvent guard with
11
        | None \Rightarrow true
12
        | Some eid' \Rightarrow beq_nat eid eid' (* beq_nat is equality on nat *)
13
      end in
14
      cEvent && (onExp guard).
                 Listing 2: Cog definitions (excerpt).
                                                           • • = • • = •
```

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Coq Definitions Extraction Integration



Coq Definitions : well

And the complete well-formedness check ...

```
Definition well (edges:edges_t) (n:nat) :=
 1
 2
      (* remove edges with event conditions *)
 3
      let pre_edges := filter no_edge edges in
 4
 5
      (* get the set of edge sources (nodes) *)
6
      let (pre_ids,_) := split pre_edges in
 7
8
      (* compute cycles, None is no cycle *)
9
      let pre_cycle :=
10
         map (ecc cyclic pre edges n nil) pre ids in
11
12
      (* check so all sources are free of cycles *)
13
      forallb (isNone (list nat)) pre cycle.
                  Listing 3: Well formedness check
```

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Coq Definitions Extraction Integration



Extraction

- The process of extracting executable code from Coq definitions consists in *discarding* all the *logical* contents and translating the computational definitions into the language of OCaml.
- In order to facilitate integration, the Coq types bool,list,prod are set to syntactically match the corresponding OCaml counterparts.

```
    Extract Inductive bool ⇒ "bool" ["true" "false"].
    Extract Inductive list ⇒ "list" ["[]" "(::)"].
    Extract Inductive prod ⇒ "(*)" ["(,)"].
    Extraction "Well.ml" well.
```

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Coq Definitions Extraction Integration



Extraction

A (prototype) IEC 61499 tool was developed, re-using OCaml code from our earlier work on the RTFM-core compiler. Conversions between OCaml types and Coq generated types are easily defined as sketched below:

```
(* to nat (Coq represenation) *)
 1
 2
   let rec int_to_nat = function
 3
      | 0 -> Well.0
 4
      | n -> Well.S (int_to_nat (n -1))
 5
 6
    (* to int (OCaml representation) *)
7
    let rec nat_to_int = function
8
     | Well.0 -> 0
9
      | Well.S n \rightarrow 1 + (nat_to_int n)
10
11
    (* to nat (Coq represenation) *)
12
    let ecc_to_nat ec =
13
      . . .
    (* to int (OCaml representation) *)
14
15
    let ecc_to_int ecc =
16
```

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Conclusion Future Work

Formalized

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Conclusion Future Work



Conclusion

- A formalization of IEC 61499 (subset) in Coq
- Liveness defined in terms of ECC scheduling progress
- A necessary and sufficient condition is defined
 - Complex (and may not be what you want)
- A sufficient (stronger) condition is a defined
 - Simple and useful
- Graph theoretical solution (SCC)
 - Requires inner SCC enumeration (NP complete)
- Addressed by pre-processing
 - Linear complexity (DFS)
- Encoded in Coq and extracted to OCaml, integrated in the RTFM-4FUN, proof of concept tool

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Conclusion Future Work

Future Work

- Proof of semantics, rendering fully certified code (for now only proof of algorithm termination)
- We are looking into why3 as a (simpler) alternative to Coq
- Extend well-formedness conditions to FB networks
- Formalize a real-time semantics for IEC 61499
- Ultimately certified
 - compilers and tools for IEC 61499
 - run-time systems for IEC 61499
 - ... your code here ...

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Conclusion Future Work

Coq, Basics

- Grounded in *Calculus of Inductive Constructions* (CIC) a typed λ -calculus that features:
 - polymorphism,
 - dependent types and
 - very expressive (co-)inductive types.
- Curry-Horward's isomorphism *programs-as-proofs* (CHi) In CHi, any typing relation *t* : *A* can either be seen as a value *t* of type *A*, or as *t* being a proof of the proposition *A*.
- Any type in Coq is in the set of sorts
 S = {Prop} ∪ {Type(i) | i ∈ ℕ}. The Type(0) sort represents computational types, while the Prop type represents logical propositions.
- Computational types can be *extracted* to functional programs
 → certified programs.

Conclusion Future Work



Inductive Types in Coq 1(2)

- An inductive type is introduced by a collection of constructors, each with its own arity.
- A value of an inductive type is a composition of such constructors.

As an example, natural numbers are encoded as follows:

Example (nat: inductive definition of natural numbers)

```
\begin{array}{l} \mbox{Inductive nat: Type} := \\ & \mid \mbox{0: nat} \\ & \mid \mbox{S: nat} \rightarrow \mbox{nat.} \end{array}
```

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Conclusion Future Work



Inductive Types in Coq 2(2)

- Coq automatically generates induction and recursion principles for each new inductive type.
- In Coq, functions must be provably terminating, e.g., recursive calls on structurally smaller arguments. As an example, consider the function plus that adds two natural numbers.

Example (plus: adds two natural numbers)

```
\begin{split} & \texttt{Fixpoint plus(n m:nat)}\{\texttt{struct n}:\texttt{nat}:=\\ & \texttt{match n with}\\ & \mid \texttt{0} \Rightarrow \texttt{m}\\ & \mid \texttt{S p} \Rightarrow \texttt{S (plus p m)}\\ & \texttt{end.} \end{split}
```